RDF and SPARQL: Database Foundations

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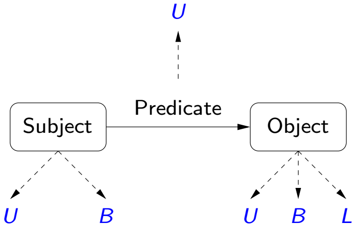
Outline

- Part I: The RDF data model
- Part II: Querying RDF Data
  - Querying: The simple and the ideal
  - Querying: Semantics and Complexity
- Part III: Querying Data with SPARQL
  - Decisions taken
  - Decisions to be taken
- Conclusions

RDF in a nutshell

- RDF is the W3C proposal framework for representing information in the Web.
- Abstract syntax based on directed labeled graph.
- Schema definition language (RDFS): Define new vocabulary (typing, inheritance of classes and properties).
- Extensible URI-based vocabulary.
- Support use of XML schema datatypes.
- Formal semantics.

RDF formal model



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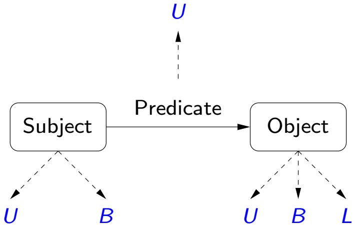

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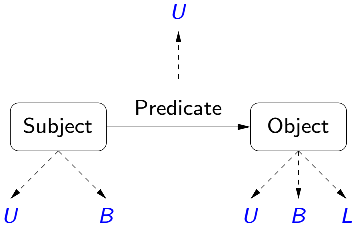

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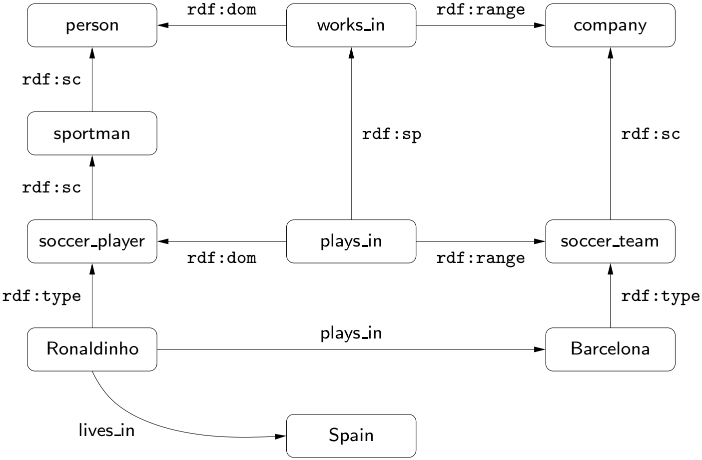
A set of RDF triples is called an RDF graph

RDFS: An example



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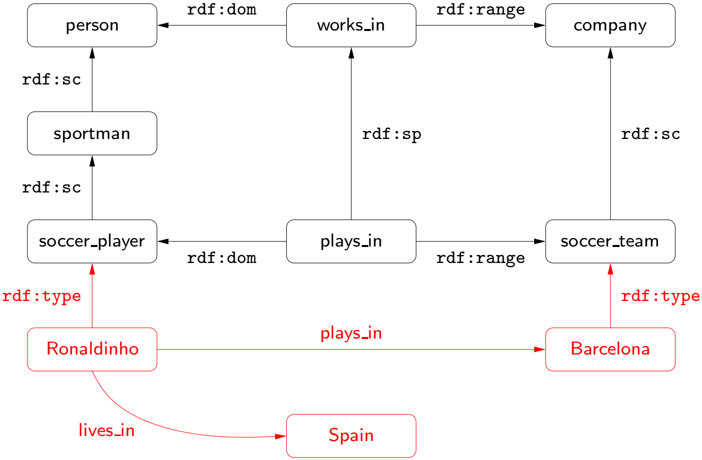

RDF model

Some difficulties:

- Existential variables as datavalues
- Built-in vocabulary with fixed semantics (RDFS)
- Graph model where nodes may also be edge labels

RDF data processing can take advantage of database techniques:

- Query processing
- Storing
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- Can be defined in terms of classical notions such model, interpretation, etc
  - As for the case of first order logic
- Has a graph characterization via homomorphisms.

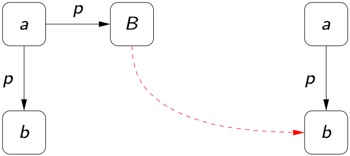
Homomorphism

A function $h : U \cup B \cup L \to U \cup B \cup L$ is a homomorphism h from G_1 to G_2 if:

- $h(c) = c$ for every $c \in U \cup L$;
- for every $(a, b, c) \in G_1$, $(h(a), h(b), h(c)) \in G_2$

Notation: $G_1 \to G_2$

Example: $h = \{B \mapsto b\}$



Entailment

Theorem (CM77)

$G_1 \models G_2$ *if and only if there is a homomorphism* $G_2 \rightarrow G_1$.

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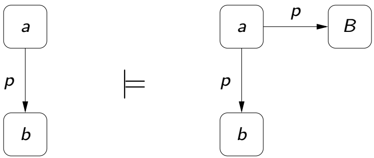

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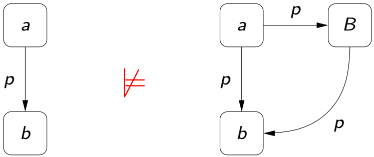

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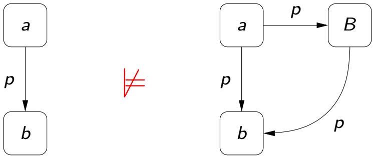

Complexity

Entailment for RDF is NP-complete

Graphs with RDFS vocabulary

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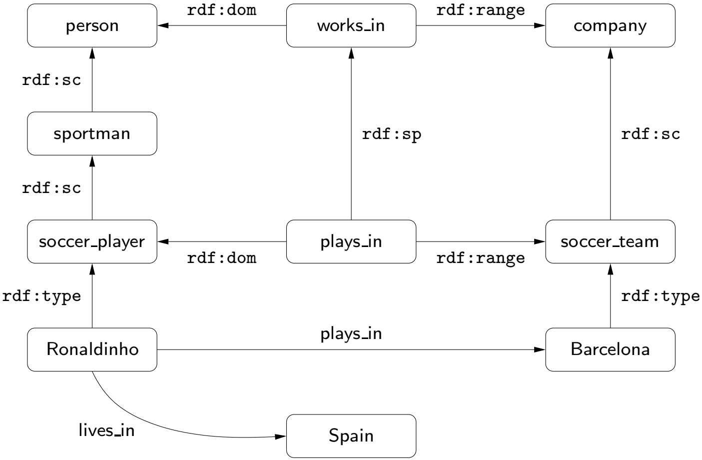

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- `rdf:sc`: transitive
- `rdf:sp`: transitive
- More complicated interactions: $\dfrac{(p, \texttt{rdf:dom}, c) \quad (a, p, b)}{(a, \texttt{rdf:type}, c)}$

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More complicated interactions: $\dfrac{(p, \texttt{rdf:dom}, c) \quad (a, p, b)}{(a, \texttt{rdf:type}, c)}$

RDFS-entailment can be characterized by a set of rules

- An Existential rule
- Subproperty rules
- Subclass rules
- Typing rules
- Implicit typing

Graphs with RDFS vocabulary: Inference rules

Inference system in [MPG07] has 14 rules:

Existential rule :

Subproperty rules :

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Implicit typing : $\dfrac{(B, \texttt{rdf:dom}, a) \quad (p, \texttt{rdf:sp}, B) \quad (b, p, c)}{(b, \texttt{rdf:type}, a)}$

RDFS Entailment

Theorem (H04,GHM04,MPG07)

$G_1 \models G_2$ iff there is a proof of G_2 from G_1 using the system of 14 inference rules.

Complexity

RDFS-entailment is NP-complete.

Proof idea

Membership in NP: If $G_1 \models G_2$, then there exists a polynomial-size proof of this fact.

Closure of an RDF Graph

Notation:

ground(G)	:	Graph obtained by replacing every blank B in G by a constant c_B.
ground^{-1}(G)	:	Graph obtained by replacing every constant c_B in G by B.

Closure of an RDF graph G (denoted by closure(G)):

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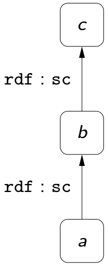
Closure of an RDF graph G (denoted by closure(G)):

$$G \cup \{t \in (U \cup B) \times U \times (U \cup B \cup L) \mid \text{there exists a ground tuple } t' \text{ such that } \text{ground}(G) \models t' \text{ and } t = \text{ground}^{-1}(t')\}$$

Closure of an RDF Graph: Example



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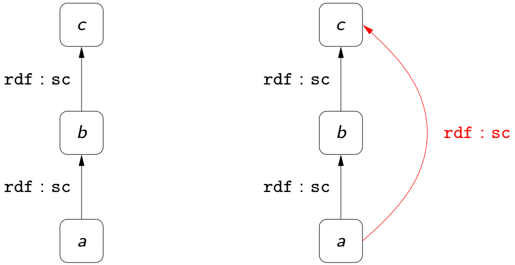

Closure of an RDF graph: complexity

Proposition (H04,GHM04,MPG07)

$G_1 \models G_2$ *iff* $G_2 \rightarrow \mathit{closure}(G_1)$

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The closure of G can be computed in time $O(|G|^4 \cdot \log |G|)$.

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The closure of G *can be computed in time* $O(|G|^4 \cdot \log |G|)$.

Can the closure be used in practice?

- Can we use an alternative materialization?
- Can we materialize a small part of the closure?

Core of an RDF Graph

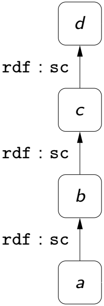
An RDF Graph G is a *core* if there is no homomorphism from G to a proper subgraph of it.

Theorem (HN92,FKP03,GHM04)

- *Each RDF graph G has a unique core (denoted by $core(G)$).*
- *Deciding if G is a core is coNP-complete.*
- *Deciding if $G = core(G')$ is DP-complete.*

Core and RDFS

For RDF graphs with RDFS vocabulary, the core of G may contain redundant information:



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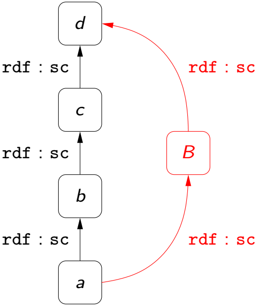

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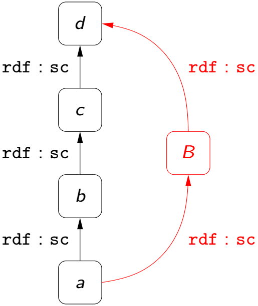

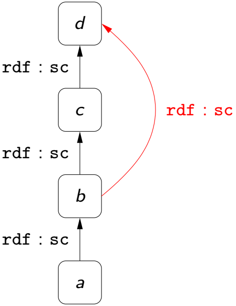

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Theorem (GHM04)

- *G_1 is equivalent to G_2 iff $nf(G_1) \cong nf(G_2)$.*
- *$G_1 \models G_2$ iff $G_2 \to nf(G_1)$*

A normal form for RDF graphs

To reduce the size of the materialization, we can combine both core and closure.

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Theorem (GHM04)

- *G_1 is equivalent to G_2 iff $nf(G_1) \cong nf(G_2)$.*
- *$G_1 \models G_2$ iff $G_2 \rightarrow nf(G_1)$*

Complexity

The problem of deciding if $G_1 = nf(G_2)$ is DP-complete.

Querying RDF data: Desiderata

Let D be a database, Q a query, and $Q(D)$ the answer.

- Outputs should belong to the same family of objects as inputs
- If $D \equiv D'$, then $Q(D) = Q(D')$
  (Weaker) If $D \equiv D'$, then $Q(D) \cong Q(D')$
- $Q(D)$ should have no (or minimal) redundancies
- The framework should be extensible to RDFS
  (Should the framework be extensible to OWL?)
- Incorporate to the framework the notion of entailment

Querying RDF data: Desiderata

Outputs should belong to the same family of objects as inputs

- Allows compositionality of queries
- Allows defining views
- Allows rewriting

In RDF, the natural objects of input/output are RDF graphs.

Querying RDF data: Desiderata

If $D \equiv D'$, then $Q(D) = Q(D')$
(Weaker) If $D \equiv D'$, then $Q(D) \cong Q(D')$

- Outputs are syntactic or semantic objects?
- Need a notion of "equivalent" databases ($\equiv$)
  (In RDF, there is a standard notion of logical equivalence)
- One could just ask logical equivalence in the output
- In RDF there is an intermediate notion: graph isomorphism

Querying RDF data: Desiderata

$Q(D)$ should have no (or minimal) redundancies

- Desirable to avoid inconsistencies
- Desirable to improve processing time and space
- Standard requirement for exchange information

Querying RDF data: Desiderata

The framework should be extensible to RDFS
(Should the framework be extensible to OWL?)

- A basic requirement of the Semantic Web Architecture
- Extension to OWL are not trivial because of the known mismatch
- Not necessarily related to the type of semantics given (logical framework, graph matching, etc.)

Querying RDF data: Desiderata

Incorporate to the framework the notion of entailment

- RDF graphs are not purely syntactic objects
- Would like to incorporate KB framework
- Beware of the complexity issues! RDF navigates on the Web
- Find the good compromise

Querying RDF data: Definitions

A conjunctive query Q is a pair of RDF graphs H, B where some resources have been replaced by variables $\bar{X}, \bar{Y}$ in V.

$$Q: \quad H(\bar{X}) \leftarrow B(\bar{X}, \bar{Y})$$

Issues:

- Free variables in B (projection)
- Treatment of blank nodes in B
- Treatment of blank nodes in H

Querying RDF data: Definitions (cont.)

A valuation is a function $v : V \to U \cup B \cup L$

A matching of a graph B in the database D is a valuation v such that $v(B) \subseteq D$.

A pre-answer to Q over D is the set

$$preans(Q, D) = \{v(H) : \ v \text{ is a matching of } B \text{ in } D\ \}$$

A single answer is an element of $preans(Q, D)$

Querying RDF data: Two semantics

Union: answer $Q(D)$ is the union of all single answers

$$ans_U(Q, D) = \bigcup preans(Q, D)$$

Merge: answer $Q(D)$ is the merge of all single answers

$$ans_M(Q, D) = \biguplus preans(Q, D)$$

Proposition

1. *For both semantics, if $D \models D'$ then $ans(Q, D) \models ans(Q, D')$*
2. *For all D, $ans_U(Q, D) \models ans_M(Q, D)$*
3. *With merge semantics, we cannot represent the identity query*

Problem

Two non-isomorphic datasets D, D' give different answers to the same query.

A slightly refined semantics:

1. Normalize D before querying
2. Then query as usual over $nf(D)$

Good News: if $D \equiv D'$ then $Q(D) \cong Q(D')$
Bad News: computing $nf(D)$ is hard

The news as formal results:

Theorem (MPG07)

Do not need to compute the normal form.

Theorem (FG06)

If a query language has the following two properties:

1. *for all Q, if $D \equiv D'$ then $Q(D) = Q(D')$,*
2. *can represent the identity query,*

then the complexity of evaluation (in data complexity) is as hard as the evaluation of $\equiv$.

Querying RDF data: Containment

A query Q' contains a query Q, denoted $Q \sqsubseteq Q'$ iff $ans(Q', D)$ comprises all the information of $ans(Q, D)$.

In classical DB: $ans(Q, D) \subseteq ans(Q', D)$

In our setting we have two versions:

- $ans(Q, D) \subseteq ans(Q', D) \quad (Q \sqsubseteq_p Q')$
- $preans(Q, D) \subseteq preans(Q', D)$ (modulo iso) $(Q \sqsubseteq_m Q')$

For ground RDF both notions coincide.

Querying RDF data: Complexity

Query complexity version: The evaluation problem is NP-complete

Data complexity version: The evaluation problem is polynomial

Querying with SPARQL

- SPARQL is the W3C candidate recommendation query language for RDF.
- SPARQL is a graph-matching query language.
- A SPARQL query consists of three parts:
  - Pattern matching: optional, union, nesting, filtering.
  - Solution modifiers: projection, distinct, order, limit, offset.
  - Output part: construction of new triples, ...

Recall the formalization from Unit-2

Syntax:

- Triple patterns: RDF triple + variables (no bnodes)
- Operators between triple patterns: AND, UNION, OPT.
- Filtering of solutions: FILTER.
- A full parenthesized algebra.

Recall the formalization from Unit-2

Semantics:

- Based on mappings, partial functions from variables to terms.
- A mapping μ is a solution of triple pattern t in G iff
  - $\mu(t) \in G$
  - $\mathrm{dom}(\mu) = \mathrm{var}(t)$.
- $[\![t]\!]_G$ is the evaluation of t in G, the set of solutions.

Example

G	t	$[\![t]\!]_G$
$(R_1$, name, john$)$ $(R_1$, email, J@ed.ex$)$ $(R_2$, name, paul$)$	$(?X$, name, $?Y)$	see below

	$?X$	$?Y$
μ_1:	R_1	john
μ_2:	R_2	paul

Compatible mappings

Definition

Two mappings are compatible if they agree in their shared variables.

Example

	$?X$	$?Y$	$?Z$	$?V$
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- μ_2 and μ_3 are not compatible

Sets of mappings and operations

Let M_1 and M_2 be sets of mappings:

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Join: $M_1 \bowtie M_2$

- extending mappings in M_1 with compatible mappings in M_2

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Difference: $M_1 \setminus M_2$

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Sets of mappings and operations

Let M_1 and M_2 be sets of mappings:

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Left Outer Join: $M_1 ⟕ M_2 = (M_1 \bowtie M_2) \cup (M_1 \smallsetminus M_2)$

Semantics of general graph patterns

Definition

Given a graph G the evaluation of a pattern is recursively defined

- $[\![(P_1 \text{ AND } P_2)]\!]_G = [\![P_1]\!]_G \bowtie [\![P_2]\!]_G$
- $[\![(P_1 \text{ UNION } P_2)]\!]_G = [\![P_1]\!]_G \cup [\![P_2]\!]_G$
- $[\![(P_1 \text{ OPT } P_2)]\!]_G = [\![P_1]\!]_G ⟕ [\![P_2]\!]_G$
- $[\![(P \text{ FILTER } R)]\!]_G = \{\mu \in [\![P]\!]_G \mid \mu \text{ satisfies } R\}$

Differences with Relational Algebra / SQL

- Not a fixed output schema
  - mappings instead of tables
  - schema is implicit in the domain of mappings
- Too many NULLs
  - mappings with disjoint domains can be joined
  - mappings with distinct domains in output solutions
- SPARQL-to-SQL translations experience these issues
  - need of `IS NULL`/`IS NOT NULL` in join/outerjoin conditions
  - need of `COALESCE` in constructing output schema

SPARQL complexity: the evaluation problem

Input:

A mapping μ, a graph pattern P, and an RDF graph G.

Question:

Is the mapping in the evaluation of the pattern against the graph?

$$\mu \in [\![P]\!]_G ?$$

Evaluation of AND-FILTER patterns is polynomial.

Theorem (PAG06)

For patterns using only AND and FILTER operators, the evaluation problem is polynomial:

$$O(|P| \times |G|).$$

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Theorem (PAG06)

For patterns using only AND *and* FILTER *operators, the evaluation problem is polynomial:*

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Proof idea

- *Check that the mapping makes every triple to match.*
- *Then check that the mapping satisfies the* FILTERs.

Evaluation including UNION is NP-complete.

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Evaluation including UNION is NP-complete.

Theorem (PAG06)

For patterns using AND, FILTER and UNION operators, the evaluation problem is NP-complete.

Proof idea

- *Reduction from 3SAT.*
- *A pattern encodes the propositional formula.*
- $\neg$ bound *is used to encode negation.*

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Proof idea

- *Reduction from QBF*
- *A pattern encodes a quantified propositional formula:*

$$\forall x_1 \exists y_1 \forall x_2 \exists y_2 \cdots \psi.$$

- *nested OPTs are used to encode quantifier alternation.*
  (This time, we do not need $\neg$ *bound.)*

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PSPACE-hardness: A closer look

Assume $\varphi = \forall x_1 \exists y_1\, \psi$, where $\psi = (x_1 \vee \neg y_1) \wedge (\neg x_1 \vee y_1)$.

We generate G, P_φ and μ_0 such that μ_0 belongs to the answer of P_φ over G iff φ is valid:

G :

P_ψ :

P_φ :

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$G \quad : \quad \{(a, \mathtt{tv}, 0),\ (a, \mathtt{tv}, 1),\ (a, \mathtt{false}, 0),\ (a, \mathtt{true}, 1)\}$

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$P_\varphi \quad :$

$\mu_0 \quad :$

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P_\varphi & : & (a, \mathtt{true}, ?B_0)\ \text{OPT}\ (P_1\ \text{OPT}\ (Q_1\ \text{AND}\ P_\psi)) \\
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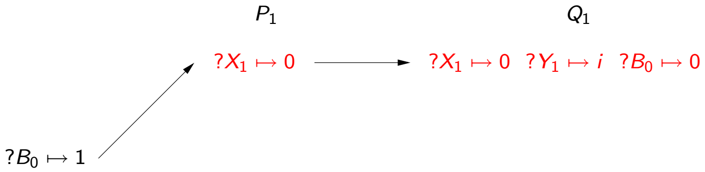

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 & & P_1 & & Q_1 \\
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Data–complexity is polynomial

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When patterns are consider to be fixed (data complexity), the evaluation problem is in LOGSPACE.

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Proof idea

From data–complexity of first–order logic.

SPARQL reordering/optimization: a simple normal from

- AND and UNION are commutative and associative.
- AND, OPT, and FILTER distribute over UNION.

Theorem (UNION Normal Form)

Every graph pattern is equivalent to one of the form

$$P_1 \ \text{UNION} \ P_2 \ \text{UNION} \ \cdots \ \text{UNION} \ P_n$$

where each P_i is UNION*–free.*

We concentrate in UNION-free patterns.

Well–designed patterns

Definition

A graph pattern is well–designed iff for every OPT in the pattern

$$(\cdots\cdots\cdots\cdots \quad (\quad A \quad \mathrm{OPT} \quad B \quad) \quad \cdots\cdots\cdots\cdots)$$

if a variable occurs inside B and anywhere outside the OPT, then the variable must also occur inside A.

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Example

$(\; (\; (?Y, \text{name}, \text{paul}) \text{ OPT } (?X, \text{email}, ?Z) \;) \quad \text{AND} \quad (?X, \text{name}, \text{john}) \;)$

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Well–designed patterns and PSPACE-hardness

In the PSPACE-hardness reduction we use this formula:

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It is not well-designed: B_0

Well–designed patterns: reordering/optimization

For well-designed patterns

- P_1 AND $(P_2$ OPT $P_3) \equiv (P_1$ AND $P_2)$ OPT P_3
- $(P_1$ OPT $P_2)$ OPT $P_3 \equiv (P_1$ OPT $P_3)$ OPT P_2

Theorem (OPT Normal Form)

Every well–designed pattern is equivalent to one of the form

$$(\cdots(\,t_1 \text{ AND } \cdots \text{ AND } t_k\,) \text{ OPT } O_1\,)\cdots) \text{ OPT } O_n\,)$$

where each t_i is a triple pattern, and each O_j is a pattern of the same form.

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Final remarks

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