RDF and SPARQL: Two basic components of the Semantic Web

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Outline

- RDF model
- Querying RDF data
  - Conjunctive queries
  - Entailment of RDF graphs
- Graphs with RDFS vocabulary
  - Inference rules
  - Querying RDFS data: Closure, Core.
- Querying RDF Data in practice: SPARQL
  - Formal semantics for SPARQL
- Complexity of the SPARQL evaluation problem
- A procedural semantics: Well–designed patterns

Semantic Web

"The Semantic Web is an extension of the current web in which information is given well-defined meaning, better enabling computers and people to work in cooperation."

[Tim Berners-Lee et al. 2001.]

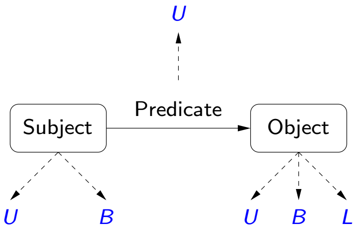
Specific Goals:

- Build a description language with standard semantics.
- Make semantics machine-processable and understandable.
- Incorporate logical infrastructure to reason about resources.
- W3C Proposal: Resource Description Framework (RDF).

RDF in a nutshell

- RDF is the W3C proposal framework for representing information in the Web.
- Abstract syntax based on directed labeled graph.
- Schema definition language (RDFS): Define new vocabulary (typing, inheritance of classes and properties).
- Extensible URI-based vocabulary.
- Support use of XML schema datatypes.
- Formal semantics.

RDF formal model

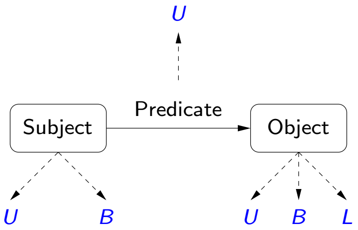


U = set of Uris

B = set of Blank nodes

L = set of Literals

RDF formal model



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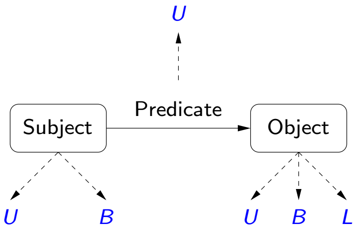

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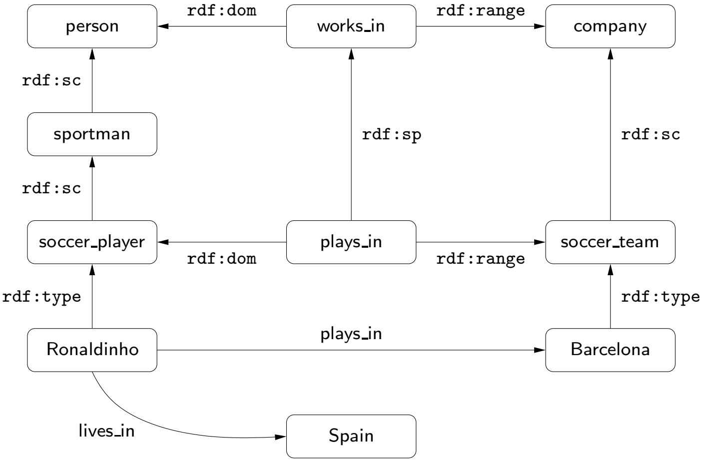
A set of RDF triples is called an RDF graph

RDFS: An example



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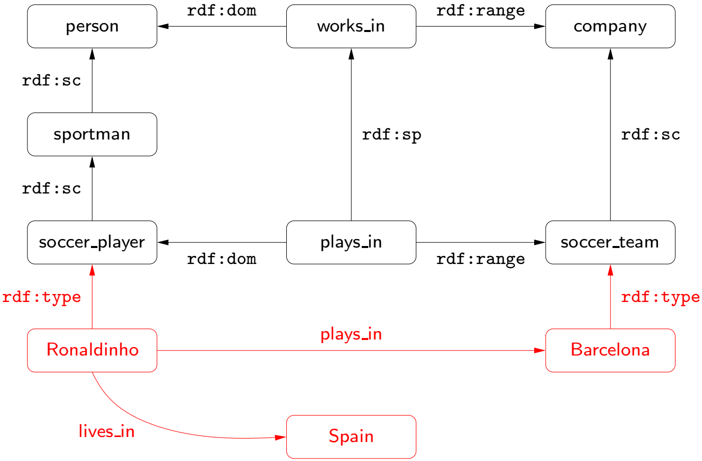

RDF model

Some difficulties:

- Existential variables as datavalues
- Built-in vocabulary with fixed semantics (RDFS)
- Graph model where nodes may also be edge labels

RDF data processing can take advantage of database techniques:

- Query processing
- Storing
- Indexing

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$$(\texttt{Ronaldinho}, \texttt{plays\_in}, X)$$

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Some examples:

$$\begin{array}{ll} & (\texttt{Ronaldinho}, \texttt{plays\_in}, \texttt{Barcelona}) \\ & (\texttt{Ronaldinho}, \texttt{plays\_in}, X) \\ \exists Y & (X, \texttt{plays\_in}, Y) \wedge (X, \texttt{lives\_in}, \texttt{Spain}) \end{array}$$

Semantics of conjunctive queries

Given an RDF graph G, a conjunctive query $Q(\overline{X})$ and a tuple $\bar{a}$ of values in $U \cup B \cup L$:

Is $\bar{a}$ an answer to $Q(\overline{X})$ in G?

Notation: $G \models Q(\bar{a})$

Notice that $Q(\overline{X})$ and $\bar{a}$ may include blank nodes.

- Blank nodes play a similar role as existential variables.
- $(\texttt{Ronaldinho}, \texttt{plays\_in}, B)$ and
  $\exists X\, (\texttt{Ronaldinho}, \texttt{plays\_in}, X)$ are *equivalent*.

Conjunctive queries and entailment of RDF graphs

$Q(\bar{a})$ can be transformed into an RDF graph G'.

- Notion to define: $G \models G'$

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Entailment of RDF graphs:

- Can be defined in terms of classical notions such model, interpretation, etc
  - As for the case of first order logic
- Has a graph characterization via homomorphisms.

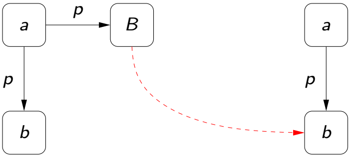
Homomorphism

A function $h : U \cup B \cup L \rightarrow U \cup B \cup L$ is a homomorphism h from G_1 to G_2 if:

- $h(c) = c$ for every $c \in U \cup L$;
- for every $(a, b, c) \in G_1$, $(h(a), h(b), h(c)) \in G_2$

Notation: $G_1 \rightarrow G_2$

Example: $h = \{B \mapsto b\}$



Entailment

Theorem (CM77)

$G_1 \models G_2$ if and only if there is a homomorphism $G_2 \rightarrow G_1$.

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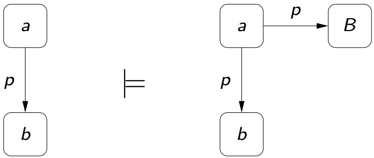

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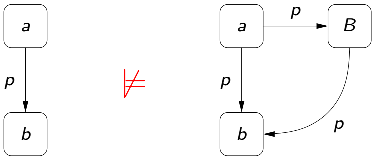

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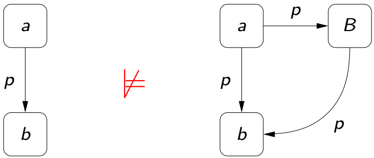

Complexity

Entailment for RDF is NP-complete

Graphs with RDFS vocabulary

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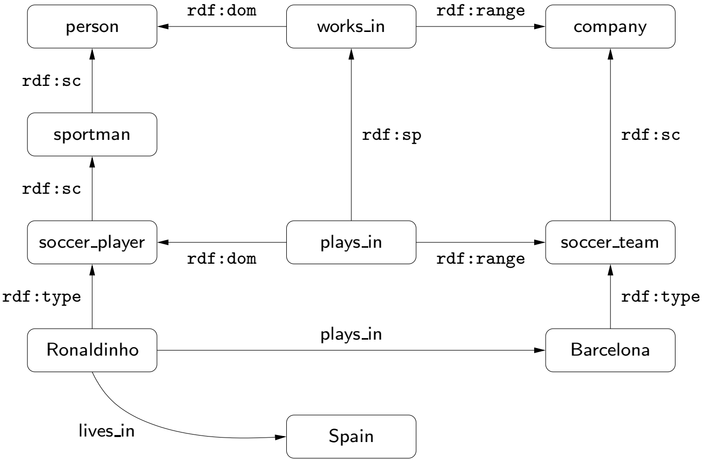

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RDFS-entailment can be characterized by a set of rules

- An Existential rule
- Subproperty rules
- Subclass rules
- Typing rules
- Implicit typing

Graphs with RDFS vocabulary: Inference rules

Inference system in [MPG07] has 14 rules:

Existential rule :

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Implicit typing : $\dfrac{(B, \texttt{rdf:dom}, a) \quad (p, \texttt{rdf:sp}, B) \quad (b, p, c)}{(b, \texttt{rdf:type}, a)}$

RDFS Entailment

Theorem (H03,GHM04,MPG07)

$G_1 \models G_2$ iff there is a proof of G_2 from G_1 using the system of 14 inference rules.

Complexity

RDFS-entailment is NP-complete.

Proof idea

Membership in NP: If $G_1 \models G_2$, then there exists a polynomial-size proof of this fact.

Querying RDFS data

System of inference rules can be used as a mechanism for evaluating queries.

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Is there any practical mechanism for evaluating queries?

- Making explicit the implicit information.

Closure of an RDF Graph

Notation:

ground(G)	:	Graph obtained by replacing every blank B in G by a constant c_B.
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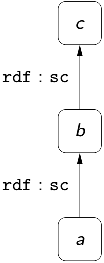
$$G \cup \{t \in (U \cup B) \times U \times (U \cup B \cup L) \mid \text{there exists a ground tuple } t' \text{ such that } \text{ground}(G) \models t' \text{ and } t = \text{ground}^{-1}(t')\}$$

Closure of an RDF Graph: Example



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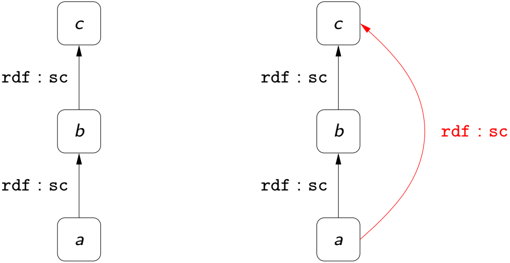

Querying RDFS data: Using the closure of a graph

Proposition (H03,GHM04,MPG07)

$G_1 \models G_2$ *iff* $G_2 \rightarrow \mathit{closure}(G_1)$

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The closure of G can be computed in time $O(|G|^4 \cdot \log |G|)$.

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Can the closure be used in practice?

- Can we use an alternative materialization?
- Can we materialize a small part of the closure?

Core of an RDF Graph

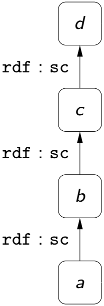
An RDF Graph G is a *core* if there is no homomorphism from G to a proper subgraph of it.

Theorem (HN92,FKP03,GHM04)

- *Each RDF graph G has a unique core (denoted by $core(G)$).*
- *Deciding if G is a core is coNP-complete.*
- *Deciding if $G = core(G')$ is DP-complete.*

Core and RDFS

For RDF graphs with RDFS vocabulary, the core of G may contain redundant information:



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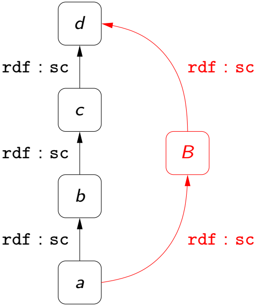

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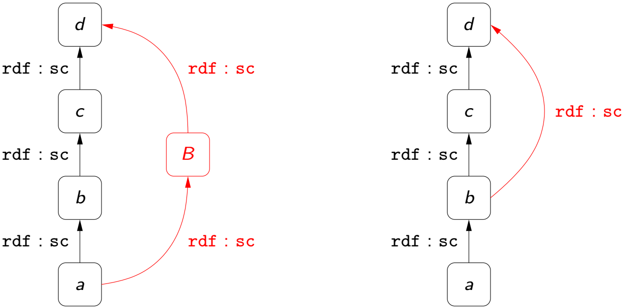

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- *$G_1 \models G_2$ iff $G_2 \rightarrow nf(G_1)$*

Complexity

The problem of deciding if $G_1 = nf(G_2)$ is DP-complete.

Querying RDF Data in practice

- SPARQL is the W3C candidate recommendation query language for RDF.
- SPARQL is a graph-matching query language.
- A SPARQL query consists of three parts:
  - Pattern matching: optional, union, nesting, filtering.
  - Solution modifiers: projection, distinct, order, limit, offset.
  - Output part: construction of new triples, ...

A simple RDF query language

```
SELECT ?Name ?Email
WHERE
{
  ?X :name ?Name
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We focus on P.

But things can become more complex ...

Interesting features of pattern matching on graphs

- Grouping
- Optional parts
- Nesting
- Union of patterns
- Filtering
- ...

```
{ P1
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    OPTIONAL { P5 }  }

  { P3
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    OPTIONAL { P7 }  }

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In our work:

- A formal compositional semantics (for simple RDF)
- Complexity bounds
- Optimization procedures

A standard algebraic syntax

- Triple patterns: just triples + variables, without blanks

original SPARQL syntax	algebraic syntax
`?X :name "john"`	$(?X, \text{name}, \text{john})$

- Graph patterns: full parenthesized algebra

original SPARQL syntax	algebraic syntax
`{ P1 P2 }`	$(P_1 \text{ AND } P_2)$
`{ P1 OPTIONAL { P2 }}`	$(P_1 \text{ OPT } P_2)$
`{ P1 } UNION { P2 }`	$(P_1 \text{ UNION } P_2)$
`{ P1 FILTER ( R ) }`	$(P_1 \text{ FILTER } R)$

original SPARQL syntax algebraic syntax

A standard algebraic syntax

- Explicit precedence/association

Example

```
{ t1
  t2
  OPTIONAL { t3 }
  OPTIONAL { t4 }
  t5
}
```

$$((((\, t_1 \text{ AND } t_2 \,) \text{ OPT } t_3 \,) \text{ OPT } t_4 \,) \text{ AND } t_5 \,)$$

Mappings: building block for the semantics

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A mapping is a partial function from variables to RDF terms.

The evaluation of a pattern results in a set of mappings.

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The semantics of triple patterns

Given an RDF graph and a triple pattern t

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Example

graph	triple	evaluation
$(R_1$, name, john$)$ $(R_1$, email, J@ed.ex$)$ $(R_2$, name, paul$)$	$(?X$, name, $?Y)$	see table below

	$?X$	$?Y$
μ_1:	R_1	john
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Compatible mappings

Definition

Two mappings are compatible if they agree in their shared variables.

Example

	?X	?Y	?Z	?V
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μ_2 :	R_1		J@edu.ex	
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- μ_2 and μ_3 are not compatible

Sets of mappings and operations

Let M_1 and M_2 be sets of mappings:

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Join: $M_1 \bowtie M_2$

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Definition

Left Outer Join: $M_1 ⟕ M_2 = (M_1 \bowtie M_2) \cup (M_1 \smallsetminus M_2)$

Semantics of SPARQL operators

Let M_1 and M_2 be the result of evaluating P_1 and P_2.

Definition

The evaluation of:

$$\begin{array}{rcl}
(P_1 \text{ AND } P_2) & \longrightarrow & \\
(P_1 \text{ UNION } P_2) & \longrightarrow & \\
(P_1 \text{ OPT } P_2) & \longrightarrow &
\end{array}$$

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Simple example

Example

$$(R_1, \text{name}, \text{john})$$
$$(R_1, \text{email}, \text{J@ed.ex})$$
$$(R_2, \text{name}, \text{paul})$$

$$((?X, \text{ name}, ?Y) \text{ OPT } (?X, \text{ email}, ?E))$$

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$(R_1, \text{email}, \text{J@ed.ex})$
$(R_2, \text{name}, \text{paul})$

$$((?X,\ \text{name},\ ?Y)\ \textcolor{red}{\text{OPT}}\ (?X,\ \text{email},\ ?E))$$

$?X$	$?Y$
R_1	john
R_2	paul

$?X$	$?E$
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- from the Join

Simple example

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R_1	J@ed.ex

- from the Difference

Simple example

Example

$(R_1, \text{name}, \text{john})$
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- from the Union

Boolean filter expressions (value constraints)

In filter expressions we consider

- equality $=$ among variables and RDF terms
- unary predicate bound
- boolean combinations ($\wedge$, $\vee$, $\neg$)

Satisfaction of value constraints

A mapping satisfies

- $?X = c$ if it gives the value c to variable $?X$
- $?X = ?Y$ if it gives the same value to $?X$ and $?Y$
- bound($?X$) if it is defined for $?X$

Definition

Evaluation of (P FILTER R): Set of mappings in the evaluation of P that satisfy R.

Natural algebraic properties: A simple normal from

- AND and UNION are commutative and associative.
- AND, OPT, and FILTER distribute over UNION.

Theorem (UNION Normal Form)

Every graph pattern is equivalent to one of the form

$$P_1 \ \text{UNION} \ P_2 \ \text{UNION} \ \cdots \ \text{UNION} \ P_n$$

where each P_i is UNION–free.

The evaluation problem

Input:

A mapping, a graph pattern, and an RDF graph.

Question:

Is the mapping in the evaluation of the pattern against the graph?

Evaluation of simple patterns is polynomial.

Theorem (PAG06)

For patterns using only AND *and* FILTER *operators, the evaluation problem is polynomial:*

$$O(\textit{size of the pattern} \times \textit{size of the graph}).$$

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Proof idea

- *Check that the mapping makes every triple to match.*
- *Then check that the mapping satisfies the* FILTERs*.*

Evaluation including UNION is NP-complete.

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- *Reduction from 3SAT.*
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- $\neg$ bound *is used to encode negation.*

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Proof idea

- *Reduction from QBF*
- *A pattern encodes a quantified propositional formula:*

$$\forall x_1 \exists y_1 \forall x_2 \exists y_2 \cdots \psi.$$

- *nested* OPT*s are used to encode quantifier alternation.*

  (This time, we do not need $\neg$ *bound.)*

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PSPACE-hardness: A closer look

Assume $\varphi = \forall x_1 \exists y_1\, \psi$, where $\psi = (x_1 \vee \neg y_1) \wedge (\neg x_1 \vee y_1)$.

We generate G, P_φ and μ_0 such that μ_0 belongs to the answer of P_φ over G iff φ is valid:

G :

P_ψ :

P_φ :

μ_0 :

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G : $\{(a, \texttt{tv}, 0),\ (a, \texttt{tv}, 1),\ (a, \texttt{false}, 0),\ (a, \texttt{true}, 1)\}$

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P_ψ : $((a, \texttt{true}, ?X_1)$ UNION $(a, \texttt{false}, ?Y_1))$ AND $((a, \texttt{false}, ?X_1)$ UNION $(a, \texttt{true}, ?Y_1))$

P_φ :

μ_0 :

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G & : & \{(a, \texttt{tv}, 0),\ (a, \texttt{tv}, 1),\ (a, \texttt{false}, 0),\ (a, \texttt{true}, 1)\} \\
 & & \\
P_\psi & : & ((a, \texttt{true}, ?X_1)\ \text{UNION}\ (a, \texttt{false}, ?Y_1))\ \text{AND} \\
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P_\varphi & : & (a, \texttt{true}, ?B_0)\ \text{OPT}\ (P_1\ \text{OPT}\ (Q_1\ \text{AND}\ P_\psi)) \\
 & & \\
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\mu_0 & : & \{?B_0 \mapsto 1\}
\end{array}
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P_1

$?X_1 \mapsto 0$

$?B_0 \mapsto 1$

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P_1 $\qquad\qquad$ Q_1

$?X_1 \mapsto 0 \longrightarrow ?X_1 \mapsto 0 \quad ?Y_1 \mapsto i \quad ?B_0 \mapsto 0$

$?B_0 \mapsto 1 \nearrow$

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$$
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 & \color{red}{?X_1 \mapsto 1} \arrow[r] & \color{red}{?X_1 \mapsto 1 \quad ?Y_1 \mapsto j \quad ?B_0 \mapsto 0}
\end{tikzcd}
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Data–complexity is polynomial

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When patterns are consider to be fixed (data complexity), the evaluation problem is in LOGSPACE.

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Proof idea

From data–complexity of first–order logic.

A procedural semantics

Suggestion of the W3C to evaluate query $A \,\text{OPT}\,(B \,\text{OPT}\, C)$:

> First compute the mappings that match A, then check which of these mappings match B, and for those who match B check whether they also match C.

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Depth–first traversal of queries parse trees.

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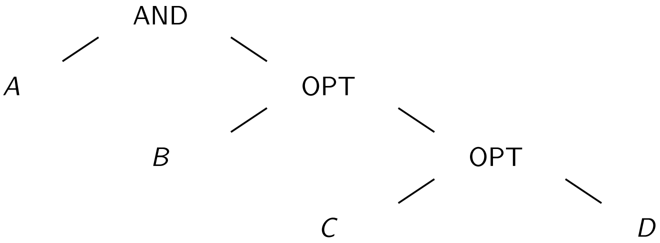
- As opposed to the bottom-up evaluation induced by the compositional semantics.

A procedural semantics

Consider: $(A \text{ AND } (B \text{ OPT } (C \text{ OPT } D)))$



- Algebraic semantics: induces the usual bottom–up evaluation.
- Alternative semantics: depth–first traversal of the parse tree.
  - Similar to the procedural semantics of Jena/ARQ
  - Navigational semantics of nested OPTs in official SPARQL (April 2006)
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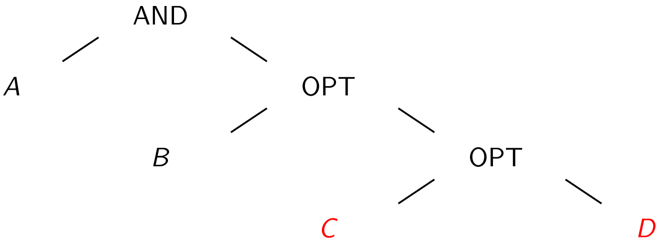

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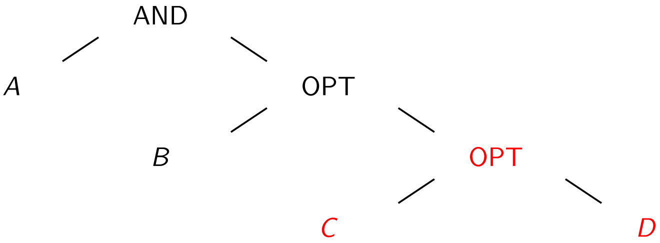

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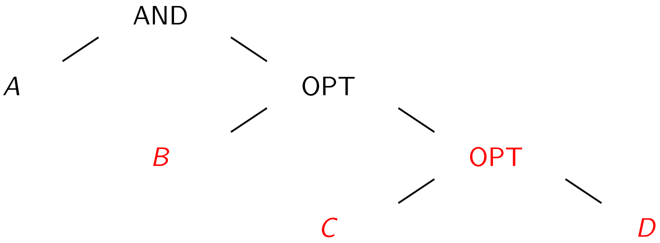

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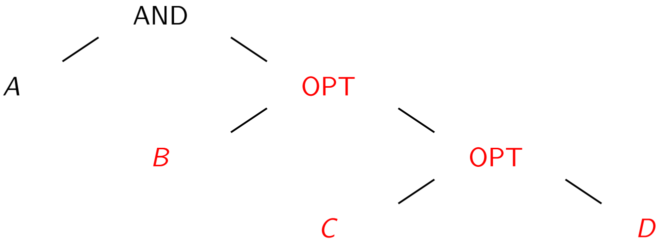

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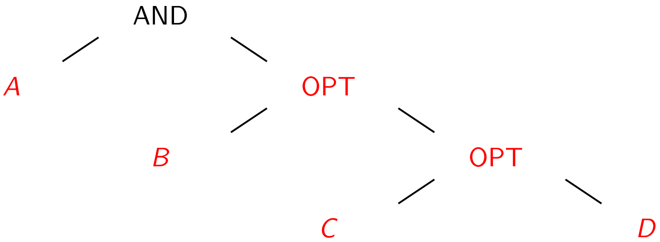

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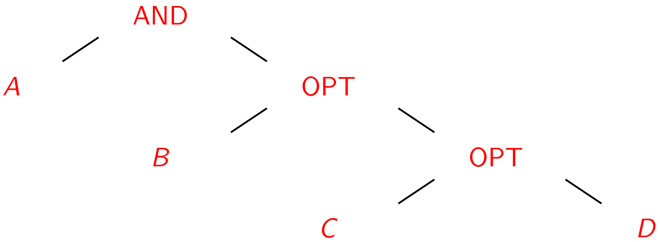

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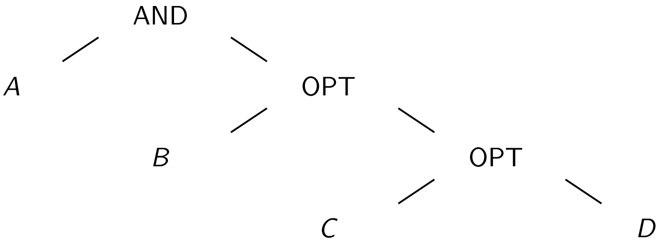

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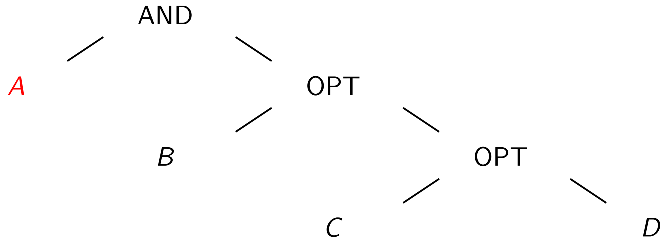

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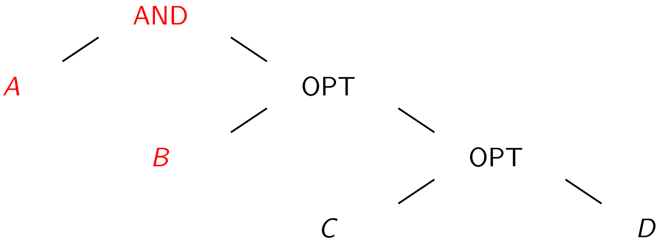

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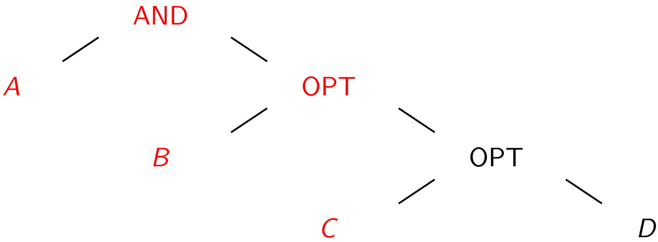

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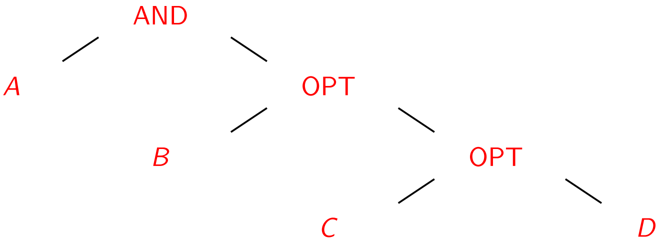

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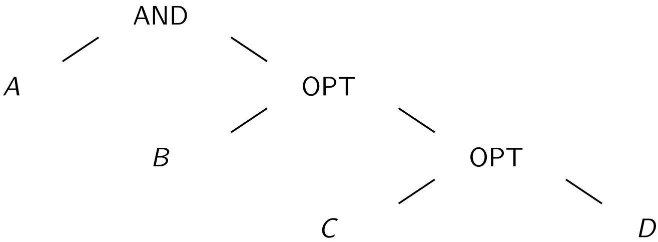

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- non-compositional
- AND of patterns is non-commutative

Well–designed patterns

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A graph pattern is well–designed iff for every OPT in the pattern

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In the PSPACE-hardness reduction we use this formula:

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It is not well-designed: B_0

Well–designed patterns

Theorem (PAG06)

For well–designed graph patterns:

$$\textit{depth–first traversal evaluation} \;=\; \textit{compositional semantics}$$

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