Locally Consistent Transformations and Query Answering in Data Exchange

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- Data Exchange Setting: $(\mathbf{S}, \mathbf{T}, \Sigma_{st})$
- S: Source schema.
- **T**: Target schema.
- Σ_{st} : Set of source-to-target dependencies.
 - Source-to-target dependency: FO sentence of the form

 $\forall \bar{x} \, (\varphi_{\mathbf{S}}(\bar{x}) \to \exists \bar{y} \, \psi_{\mathbf{T}}(\bar{x}, \bar{y})).$

- $\varphi_{\mathbf{S}}(\bar{x})$: FO formula over S.
- $\psi_{\mathbf{T}}(\bar{x}, \bar{y})$: conjunction of FO atomic formulas over \mathbf{T} .

Data exchange settings: Example

 $\mathbf{S} = \langle Employee(\cdot) \rangle$

 $\mathbf{T} = \langle Dept(\cdot, \cdot) \rangle$

 $\Sigma_{st} = \{ \forall x \, (Employee(x) \to \exists y \, Dept(x, y)) \}.$

2

Given a source instance I, find a target instance J such that (I, J) satisfies Σ_{st} .

- *J* is called a solution for *I*.

Example: Possible solutions for $I = \{Employee(peter)\}$:

- $J_1 = \{Dept(peter, 1)\}.$
- $J_2 = \{Dept(peter, 1), Dept(peter, 2)\}.$
- $J_3 = \{Dept(peter, 1), Dept(john, 1)\}.$
- $J_4 = \{Dept(peter, X)\}.$
- $J_5 = \{Dept(peter, \mathbf{X}), Dept(peter, \mathbf{Y})\}.$

- Q: Query over the target schema.
 - What does it mean to answer Q?

$$\underline{certain}(Q, I) = \bigcap_{J \text{ is a solution for } I} Q(J)$$

Example:

- <u>certain</u>($\exists y \ Dept(x, y), I$) = {peter}.
- <u>certain</u>(Dept(x, y), I) = \emptyset .

Query rewriting

How can we compute $\underline{certain}(Q, I)$?

- Naïve algorithm does not work: infinitely many solutions.

Approach proposed in [FKMP03]: Query Rewriting

Look for some specific \mathcal{F} : $inst(\mathbf{S}) \rightarrow inst(\mathbf{T})$, and find conditions under which $\underline{certain}(Q, I) = Q'(\mathcal{F}(I))$ for every source instance I.

What is a good alternative for \mathcal{F} ?

Outline

- Query rewriting over the canonical solution.
- Locality in data exchange.
 - Proving inexpressibility results.
- Query rewriting over the core.
 - Canonical solution versus core.
- Extensions.
 - Other semantics.
- Conclusions.

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Canonical solution

Input: $(\mathbf{S}, \mathbf{T}, \Sigma_{st})$ and a source instance I

Output: Canonical solution J for I

Algorithm:

for every $\forall \bar{x} (\varphi_{\mathbf{S}}(\bar{x}) \to \exists y \psi_{\mathbf{T}}(\bar{x}, \bar{y})) \in \Sigma_{st}$ do for every \bar{a} such that I satisfies $\varphi_{\mathbf{S}}(\bar{a})$ do create a fresh tuple of null values \overline{X} insert $\psi_{\mathbf{T}}(\bar{a}, \overline{X})$ into J

$$\Sigma_{st} = \{ \forall x \, (Employee(x) \to \exists y \, Dept(x, y)) \} \text{ and } I = \{ Employee(peter), \, Employee(john) \}.$$

- For $a = peter \operatorname{do}$

Create a fresh null value X

Insert Dept(peter, X) into J

- For a = john do

Create a fresh null value YInsert Dept(john, Y) into J

Canonical solution:

 ${Dept(peter, X), Dept(john, Y)}$

Query rewriting over the canonical solution

 $\mathcal{F}_{\operatorname{can}}(I)$: canonical solution for *I*.

- Can be computed in polynomial time (data complexity).

Theorem [FKMP03]: For every data exchange setting and union of conjunctive queries Q, there exists Q' such that for every source instance I, <u>certain</u> $(Q, I) = Q'(\mathcal{F}_{can}(I))$.

- C(x): holds whenever x is a constant.
- $Q'(x_1,\ldots,x_m) = C(x_1) \wedge \cdots \wedge C(x_m) \wedge Q(x_1,\ldots,x_m).$

Query rewriting over the canonical solution

Can the theorem be extended to other classes of queries?

Theorem [FKMP03]: There exists a data exchange setting and a conjunctive query Q with one inequality such that Q is not FO-rewritable over \mathcal{F}_{can} .

- For every FO query Q', there exists an instance I such that $\underline{certain}(Q, I) \neq Q'(\mathcal{F}_{can}(I)).$

We would like to study the query rewriting problem.

- We need some tools: How can we prove that a query is not FO-rewritable?

Query rewriting: Some facts

The problem of deciding whether an FO formula is FO-rewritable over \mathcal{F}_{can} is undecidable.

There exists other classes of queries that are FO-rewritable over the canonical solution.

- Every boolean query Q whose asymptotic probability is 0 is FO-rewritable: $\underline{certain}(Q, I) = false$.

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Locality in data exchange: Notation

I: source instance.

Gaifman graph $\mathcal{G}(I)$ of I:

- $\operatorname{adom}(I)$ is the set of nodes of $\mathcal{G}(I)$.
- There exists an edge between a and b iff a and b belong to the same tuple of a relation in I.

Example: $I(R) = \{(1, 2, 3)\}$ and $I(T) = \{(1, 4), (4, 5)\}.$



 $d_I(a, b)$: distance between a and b in $\mathcal{G}(I)$.

 $d_I(\bar{a}, b)$: minimum value of $d_I(a, b)$, where a is in \bar{a} .

 $N_d^I(\bar{a})$: restriction of I to the elements at distance at most d from \bar{a} .

- Example: $\operatorname{adom}(N_2^I(5)) = \{1, 4, 5\}, N_2^I(5)(R) = \emptyset$ and $N_2^I(5)(T) = \{(1, 4), (4, 5)\}.$

 $N_d^I(\bar{a}) \cong N_d^I(\bar{b})$: members of \bar{a} and \bar{b} are treated as distinguished elements.

-
$$\bar{a} = (a_1, ..., a_m)$$
 and $\bar{b} = (b_1, ..., b_m)$.

- There is an isomorphism $f: N_d^I(\bar{a}) \to N_d^I(\bar{b})$ such that $f(a_i) = b_i$ $(1 \le i \le m).$ Locality in data exchange: Definition

Given: $(\mathbf{S}, \mathbf{T}, \Sigma_{st})$ and *m*-ary query *Q* over **T**.

Definition: Q is **locally source-dependent** if there is $d \ge 0$ such that for every instance I of S and m-tuples \bar{a}, \bar{b} in I,

 $\bar{a} \in \underline{\operatorname{certain}}(Q, I)$ $N_d^I(\bar{a}) \cong N_d^I(\bar{b}) \implies \qquad \text{iff}$ $\bar{b} \in \underline{\operatorname{certain}}(Q, I)$

Locality in data exchange: Main theorem

Theorem: If Q is FO-rewritable over the canonical solution, then Q is locally source-dependent.

This theorem can be used to prove inexpressibility results.

- If a query is not locally source-dependent, then it is not FO-rewritable.

Example: Proving inexpressibility

Data exchange setting:

$$S = \langle G(\cdot, \cdot), R(\cdot), S(\cdot) \rangle$$

$$T = \langle G'(\cdot, \cdot), R'(\cdot), S'(\cdot) \rangle$$

$$\Sigma_{st} = \forall x \forall y (G(x, y) \rightarrow G'(x, y)),$$

$$\forall x (R(x) \rightarrow R'(x)),$$

$$\forall x (S(x) \rightarrow S'(x)).$$

Query:

 $Q(x) = R'(x) \lor S'(x) \land \exists y \exists z (R'(y) \land G'(y, z) \land \neg R'(z))$

Assume that Q is FO-rewritable over the canonical solution.

Then there exists $d \ge 0$ such that

 $N_d^I(a) \cong N_d^I(b) \implies a \in \underline{\operatorname{certain}}(Q, I) \text{ iff } b \in \underline{\operatorname{certain}}(Q, I).$

Contradiction: find a source instance I such that

 $N_d^I(a) \cong N_d^I(b), \ a \in \underline{certain}(Q, I) \ \text{and} \ b \notin \underline{certain}(Q, I).$

Example: Defining instance I



Example: $a \in \underline{certain}(Q, I)$

If J does not satisfy $S'(a) \land \exists y \exists z (R'(y) \land G'(y, z) \land \neg R'(z))$:



Then: J satisfies R'(a).

Example: $b \notin \underline{certain}(Q, I)$



J does not satisfy $R'(b) \vee S'(b) \wedge \exists y \exists z (R'(y) \wedge G'(y, z) \wedge \neg R'(z)).$

Example: Getting a contradiction



Conclusion: Q is **not** FO-rewritable over the canonical solution.















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What about other transformations?

Core of canonical solution J: Substructure J^* of J such that there is a homomorphism from J to J^* and there is no homomorphism from J to a proper substructure of J^* .

- Homomorphism $h: J \to J'$: mapping from adom(J) to adom(J') such that h(c) = c for all constant c, and $\bar{t} \in J(R)$ implies $h(\bar{t}) \in J'(R)$.

Core is the smallest solution that is *homomorphically equivalent* to the canonical solution.

- It can be computed in polynomial time [FKP03].

Query rewriting over the core

 $\mathcal{F}_{core}(I)$: core of the canonical solution for *I*.

Theorem [FKMP03]: For every data exchange setting and union conjunctive queries Q, there exists Q' such that for every source instance I, <u>certain</u> $(Q, I) = Q'(\mathcal{F}_{core}(I))$.

- Certain answers can be computed more efficiently by using the core.

Rewritability over the core: Can we use locality?

Canonical solution versus core: First attempt

Proposition: There exists a data exchange setting $\mathcal{A} = (\mathbf{S}, \mathbf{T}, \Sigma_{st})$ such that for every data exchange setting $\mathcal{B} = (\mathbf{S}, \mathbf{T}, \Gamma_{st})$, there exists instance I of \mathbf{S} such that:

$$\mathcal{F}^{\mathcal{A}}_{\operatorname{core}}(I) \cong \mathcal{F}^{\mathcal{B}}_{\operatorname{can}}(I).$$

We need a different approach ...

Expressiveness: Canonical solution versus core

Theorem: If Q is FO-rewritable over the core, then Q is also FO-rewritable over the canonical solution.

- There is a PTIME algorithm that, given a rewriting of Q over the core, finds a rewriting of Q over the canonical solution.

Corollary: If Q is FO-rewritable over the core, then Q is locally source-dependent.

Proof sketch

Assume $\varphi(\bar{x}) = \exists u \forall v \, \psi(\bar{x}, u, v)$ is a rewriting of Q over the core, where $\psi(\bar{x}, u, v)$ is quantifier-free.

- For every source instance I and tuple of constants \bar{a} : $\bar{a} \in \underline{certain}(Q, I)$ iff $\mathcal{F}_{core}(I) \models \varphi(\bar{a}).$

Assume that:

- $\alpha_1(x)$: holds if there is a core of $\mathcal{F}_{can}(I)$ containing null x.
- $\alpha_2(x,y)$: holds if there is a core of $\mathcal{F}_{can}(I)$ containing nulls x and y.

Proof sketch

If $\alpha_1(x)$ and $\alpha_2(x, y)$ are FO-definable, then Q is FO-rewritable over the canonical solution:

 $\bar{a} \in \underline{certain}(Q, I) \quad \text{iff} \quad \mathcal{F}_{\text{core}}(I) \models \exists u \forall v \, \varphi(\bar{a}, u, v)$ $\text{iff} \quad \mathcal{F}_{\text{can}}(I) \models \exists u \, (\alpha_1(u) \land \forall v \, (\alpha_2(u, v) \to \varphi(\bar{a}, u, v))).$

How can we define $\alpha_1(x)$ and $\alpha_2(x, y)$ in FO?

- We show how to define $\alpha_1(x)$.

Proof sketch

Notation:

If J is a canonical solution: $|\mathsf{nulls}(X, J)|$ and $|\mathsf{block}(X, J)|$ are bounded.

Lemma: Let J be the canonical solution for I and X a null value of J. There exists a core of J containing X iff for every pair of target structures J', J'' satisfying the following conditions:

- $J' \subseteq J$ and $|J'| \leq |\mathsf{block}(X, J)|$,
- there exists a homomorphism $h : block(X, J) \to J'$ such that X is not a null of h(block(X, J)),

- and
$$J' \subseteq J'' \subseteq \left(J' \cup \bigcup_{\{X \mid X \text{ is a null of } J'\}} \mathsf{block}(X, J)\right)$$
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Is this definable in FO?
- and
$$J' \subseteq J'' \subseteq \left(J' \cup \bigcup_{\{X \mid X \text{ is a null of } J'\}} block(X, J)\right),$$

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Expressiveness: Canonical solution versus core

Theorem: There exists an FO query that is FO-rewritable over the canonical solution but not over the core.

Expressiveness point of view: Canonical solution is better than the core.

- Canonical solution contains more information than the core.

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Usual certain answers semantics sometimes exhibit counterintuitive behavior.

Good solutions: Universal solutions.

- Homomorphically equivalent to the canonical solution.

May be more meaningful to consider semantics based on universal solutions:

$$\underline{u\text{-certain}}(Q, I) = \bigcap_{J \text{ is a universal solution for } I} Q(J).$$

Query rewriting under the universal solutions semantics

Given query Q, we want to find Q' such that <u>u-certain</u> $(Q, I) = Q'(\mathcal{F}(I))$ for every source instance I.

Theorem [FKP03]: For every data exchange setting and existential query Q, there exists Q' such that for every source instance I, <u>u-certain</u> $(Q, I) = Q'(\mathcal{F}_{core}(I)).$ Query rewriting under the universal solutions semantics

Definition: Q is locally source-dependent under the universal solution semantics if there is $d \ge 0$ such that:

 $\bar{a} \in \underline{u\text{-certain}}(Q, I)$ $N_d^I(\bar{a}) \cong N_d^I(\bar{b}) \implies \qquad \text{iff}$ $\bar{b} \in \underline{u\text{-certain}}(Q, I)$

Theorem: All the previous results hold for the universal solution semantics.

- If Q is FO-rewritable over the canonical solution (core) under the universal solutions semantics, then Q is locally source-dependent under the universal solutions semantics.

What about target constraints?

Locality is no longer valid.

tgd: Even with a single full tgd.

 $\forall x \forall y \forall z \, (R(x,y) \land R(y,z) \to R(x,z)).$

egd: Even for key dependencies.

Except for GAV settings: $\forall \bar{x} (\varphi_{\mathbf{S}}(\bar{x}) \to T(\bar{x})).$

Conclusions

- Common data exchange transformations map similar neighborhoods into similar neighborhoods.
- This propertity can be used to formulate a locality notion for the canonical solution and the core.
- Locality can be used to prove that a query is not FO-rewritable.
 - Holds for other semantics.
- Expressiveness point of view: Canonical solution is better than the core.